MAXWELL-LIKE RELATIONS IN CONDENSED MATERIALS. DECAY OF SHOCK WAVES†

GEORGE E. DUVALL

Physics Department, Washington State University, Pullman, WA 99164, U.S.A.

Abstract—The processes of shock wave decay are shown to include geometric attenuation, which is present for all wave amplitudes, hydrodynamic attenuation which is important only for large amplitude waves and depends only on mechanical and thermodynamic variables, and 'Maxwell attenuation' which results from the introduction of additional variables into the constitutive equations. A general shock wave decay equation is derived and illustrated by several examples. The effect of entropic flow behind the shock front is shown to modify the decay rate, but not to introduce new terms. The case of a viscous fluid is examined and shown to produce attenuation in addition to hydrodynamic attenuation. The relative importance of the new term has not been estimated.

1. INTRODUCTION

Traditional discussions of constitutive relations are fragmented and incoherent. Some progress has been made in recent years towards the development of general theories which encompass certain classes of constitutive relations[1, 2], but by and large most of us still go to thermodynamics for equations of state, fluid mechanics for the properties of viscous fluids, plasticity sources and dislocation theory for deformation, etc. One of the reasons for such fragmentation is the natural inclination of scientists to focus their attention on the subject of immediate interest, and the division of science is such that investigators interested in static phenomena, for example, are rarely concerned with real-time behavior of materials, plasticians rarely care about thermodynamic properties, etc. A major reason that this isolation has diminished in the last twenty years has been the rising interest in non-linear stress waves—principally shock waves—which dates from the period immediately following World War II.

Shock wave compressions can be very large and they are accompanied by high temperatures, so thermodynamic considerations are essential for understanding shock experiments and for using them to investigate material behavior. As soon as shock wave experimentation became sophisticated enough that wave profiles and their evolution in time could be recorded, it was evident that shock propagation phenomena resulted from a combination of thermodynamic and various irreversible, including time-dependent, properties. As a consequence, much of the effort in shock wave research during the last 15 years has been directed toward the understanding of these combined thermodynamic and non-thermodynamic effects and their use in the unravelling of various properties of materials in which they are made to propagate. A further development, as noted above, has been a deepening interest of theoreticians in the general theory of constitutive relations and their consequent involvement in the interpretation of experiments.

The central problem in which both thermodynamic and irreversible or time-dependent phenomena play a role is that of shock wave decay. The term 'decay' is used in a generic sense. Reactive or hydrodynamic processes may be such as to cause the shock amplitude to grow with time, but the equations are unchanged. A uniform shock state is useful for measurement of fundamental thermodynamic properties, but it yields no information about time-dependent properties. The steady shock transition region or 'permanent regime' contains such information, but it has proved to be of little use in analyzing the properties of solids. If, however, the shock wave profile can be recorded in a region where it is changing as it propagates, it may be possible to relate such changes to both reversible and irreversible material properties. In the most general case, both the time duration of application of the shock-producing pressure and its release may be controlled[3]; then both shock formation and shock decay can be studied. In the simple case, to which the present report is limited, pressure

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application is practically instantaneous and is followed by time-dependent release of pressure. The resulting shock wave decays as it propagates and the rate of change of shock amplitude with distance depends upon two additive terms:

$$\frac{Dp_1}{DX} = -A_1 \frac{\partial p}{\partial x} - A_2. \tag{1.1}$$

Here the shock has been assumed to be plane, with amplitude $p_1(X)$ located at $X(t)$ and to propagate in the direction of increasing x; $\partial p/\partial x$ is the gradient of shock pressure immediately behind the shock front, A_1 depends on thermomechanical parameters and A_2 depends on other, sometimes irreversible, properties of the material through which the shock is propagating, or of the system in which it is incorporated. For diverging waves a third term due to geometric attenuation appears on the right-hand side of equation (1.1).

The effect of the $\partial p/\partial x$ term has often been called "hydrodynamic attenuation", since it is entirely independent of dissipative properties in the material. Existence of the second term, A_2, has denoted what has been called a "stress-relaxing solid", "relaxing solid", "viscoelastic solid", or "Maxwell solid". In this paper the material will be called "Maxwell-like" if $A_2 \neq 0$, whatever its source. In contrast with the effect of the first term, the effect of the A_2 term will be called "Maxwell attenuation" to indicate that it depends on other than normal thermodynamic properties, i.e. stress, strain and entropy or temperature. In the following sections the natures of hydrodynamic and Maxwell attenuations will be illustrated, the coefficients A_1 and A_2 will be derived under rather general conditions and several examples will be given in which A_2 is non-vanishing.

The main objective of this paper is to demonstrate that whenever the (adiabatic) constitutive relation for the shocked state involves more than two parameters, the material will be Maxwell-like.

2. MECHANISMS OF SHOCK DECAY

Stress and strain conventions

We shall be dealing exclusively with waves in one space dimension, x. All equations will refer to mechanically-isotropic materials. Expansion to anisotropic media can be accomplished by procedures described elsewhere[4, 5]. Materials of interest are primarily solids or viscous fluids, so shear stresses commonly exist. Diagonal stress components will be exclusively compressive, so it is convenient to follow the practice of fluid dynamics and use the pressure tensor, p_{ij}, which is the negative of the stress tensor, σ_{ij}, commonly used in solid body mechanics:

$$p_{ij} = -\sigma_{ij}.$$

With the coordinate convention described above, x, y and z are principal coordinates; off-diagonal components of p_{ij} vanish, and diagonal components can be described by a single subscript; $p_x \equiv p_{xx}, p_y \equiv p_{yy}, p_z \equiv p_{zz}$. Because of the symmetry of one-dimensional waves, $p_y = p_z$. No motions parallel to wave fronts will be considered, so the only non-vanishing component of strain for plane waves is $\epsilon_x \equiv \epsilon_{xx}$; this is called a "state of uniaxial strain".

The only pressure component which is normally measured in experiments with plane shock waves is p_x, which can be looked upon as composed of mean pressure, $\bar{p}$, and a shearing stress, τ. This useful resolution is a simple identity:

$$p_x = (p_x + 2p_y)/3 + 2(p_x - p_y)/3$$

$$= \bar{p} + 4\tau/3$$

where τ is shear stress on planes with normals at 45° to the x axis. When dealing with hydrodynamic states, or when shock wave results are to be compared with static measurements, $\bar{p}$ will be identified with hydrostatic pressure P.